Aalto University
School of Engineering

How to Work with Curved Structures ?

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- Why do we need improvements?
- What have we done?
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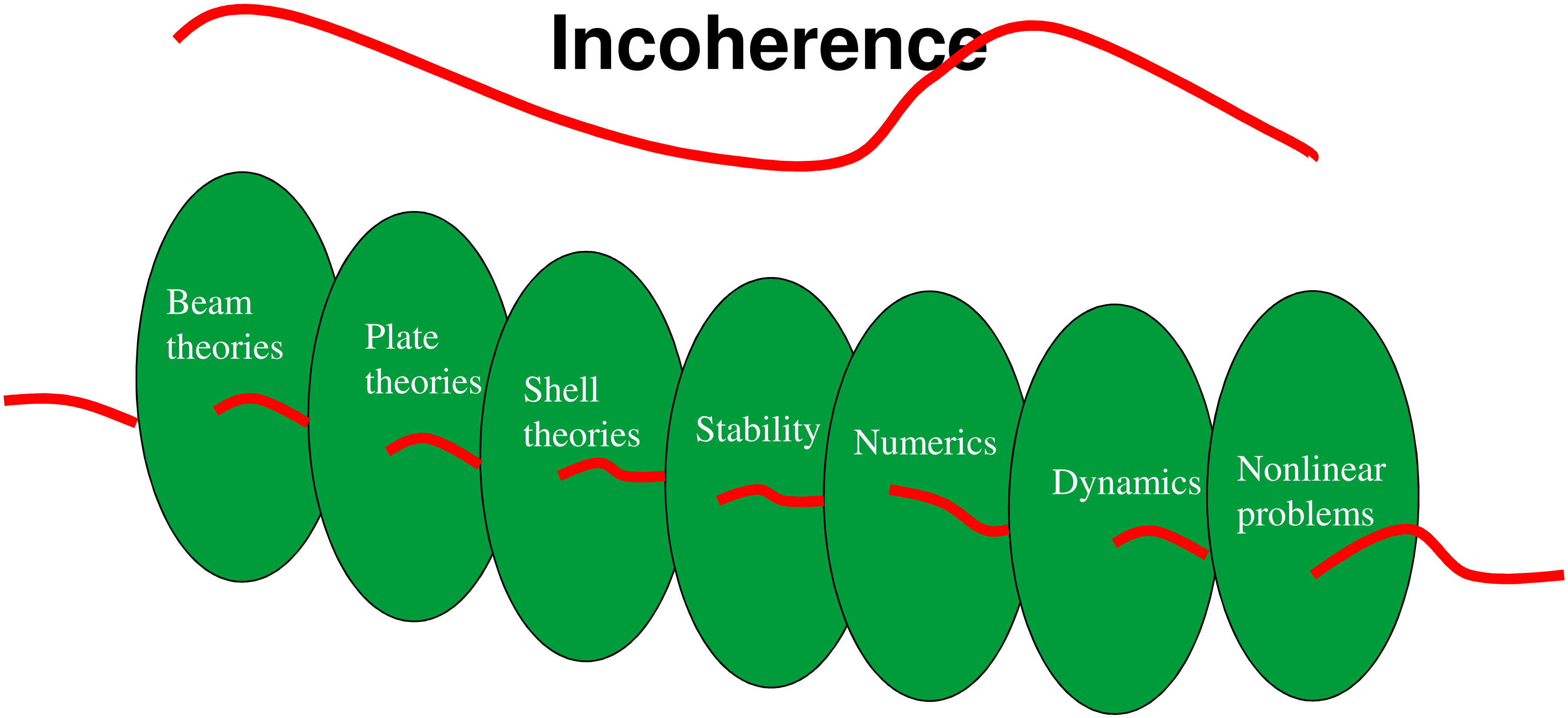
Background

- There is a strong need to develop educational processes of theoretical topics
- Computational visualization, ideas of modern pedagogy and other strategies are the most often applied improvements - the mechanics or the theories themselves have got only the minor role
- We have tried to find out some tools to help students to become inspired in the theoretical matter, which is based mathematically on rudiments, only (vector calculus)

Why have we done what we have done?

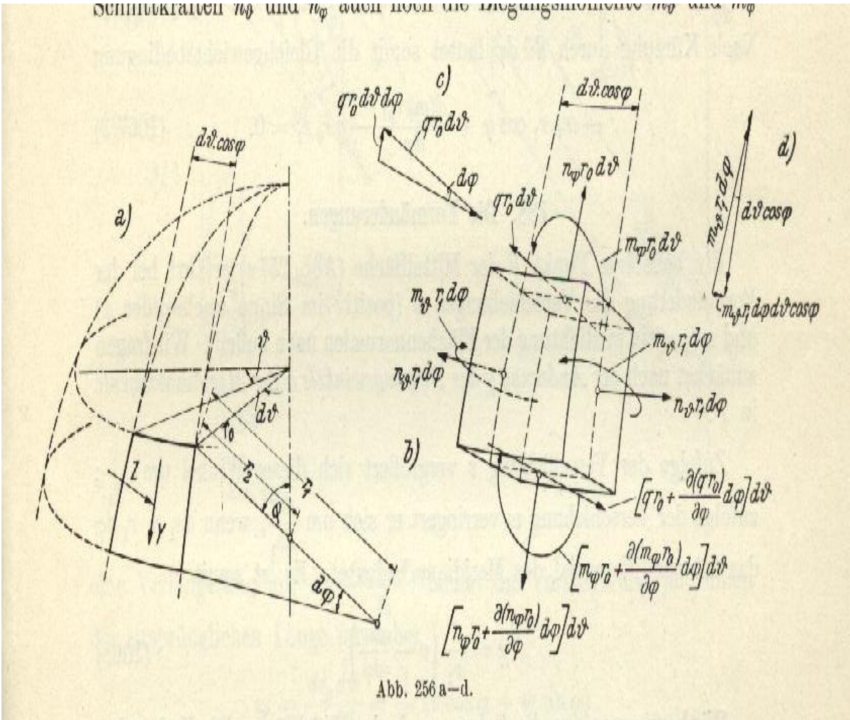
- There are basically two main reasons:

Problems of Mechanics



and secondly

The figures of differential geometry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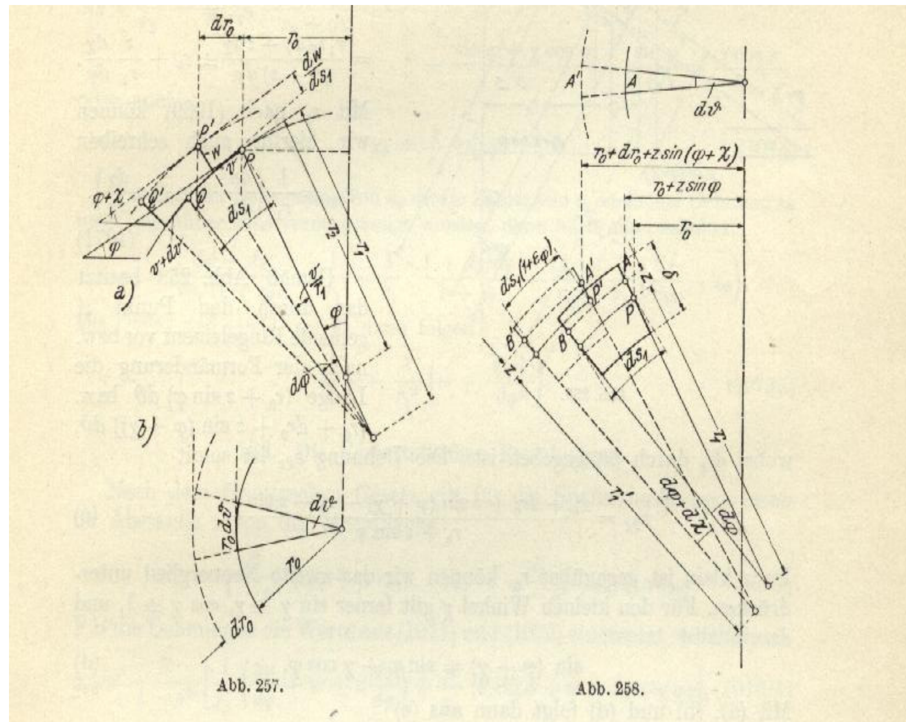
Abb. 257. Abb. 258.

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Minor challenges

- Teaching is traditionally too method-oriented
- Mechanics has a strong overlapping with Mathematics
- The fear of curved geometries

What is then needed ?

- General common mathematical tool, which is
  - based on rudiments only
  - applicable to all kind of problems
  - exact with no dubioucity
  - easy to learn

The idea for improvement

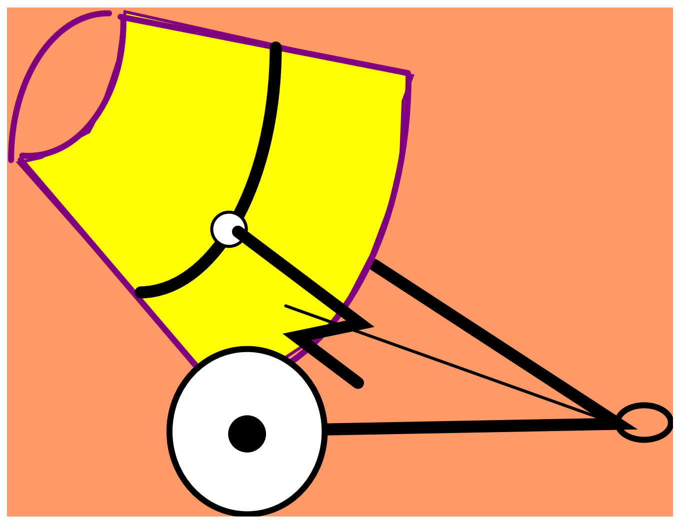
INPUT

Geometry

Kinematics

Mathematical manipulator or

Mill



OUTPUT

Equations needed, strains, equilibrium etc.

How will this be done ?

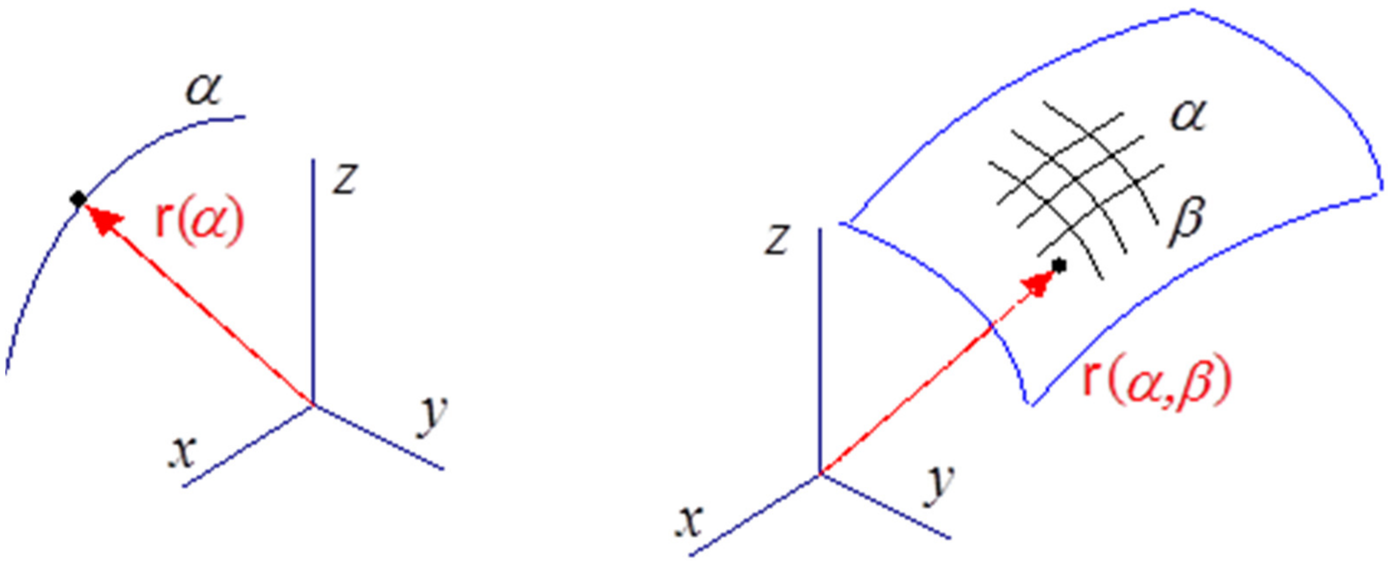
- The medicine we are serving
  - Vector calculus – defining the geometry and kinematics in vector fields
  - Use of a local Cartesian frame – a way to avoid defining various derivatives in curvilinear coordinates
  - Energy principles and principle of virtual work

Geometry description

- is given in curvilinear co-ordinates by using the position vector

$$\mathbf{r} = \mathbf{r}\{\alpha, \beta, \gamma\}$$

Position vector



Geometry

is given in curvilinear co-ordinates by using the position vector

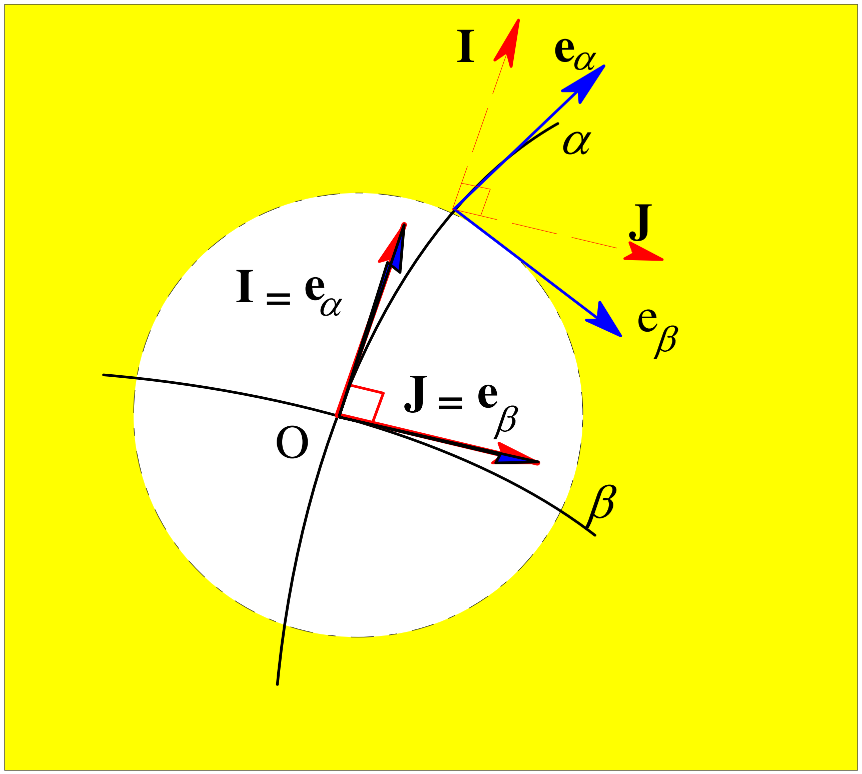
$$\mathbf{r} = \mathbf{r}\{\alpha, \beta, \gamma\}$$

mathematically it defines the domain to be considered

it defines also all the mathematical operators

$$\text{grad, div, curl, } \nabla, \Delta, \text{ etc}$$

The role of the local frame according to B. Irons

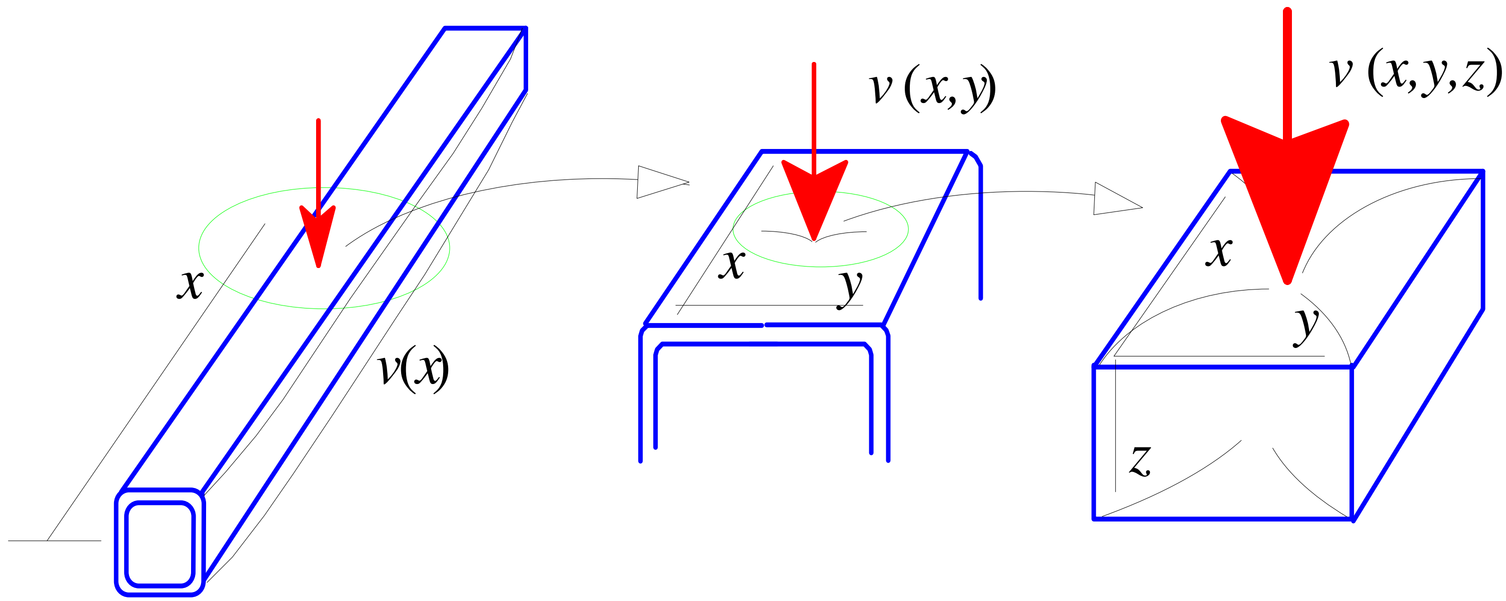


The unit vectors of local frame are constant, both in magnitude and direction

Kinematics

- Kinematics is a tool controlled by the analyst himself both in analytical and numerical analyses
- It can be interpreted as the freedom of the structure to be allowed to deform – given by the user
- It will define whether the problem is 1-, 2- or 3-dimendional

1-, 2- or 3-dimensional



Local Cartesian frame

Strains are defined in local frame – linear ones

$$\varepsilon_X = \frac{\partial \mathbf{u}}{\partial X} \bullet \mathbf{e}_X, \quad \gamma_{XY} = \frac{\partial \mathbf{u}}{\partial X} \bullet \mathbf{e}_Y + \frac{\partial \mathbf{u}}{\partial Y} \bullet \mathbf{e}_X, \quad \text{etc.}$$

or non-linear

$$\varepsilon_X = \frac{\partial \mathbf{u}}{\partial X} \bullet \mathbf{e}_X + \frac{1}{2} \frac{\partial \mathbf{u}}{\partial X} \bullet \frac{\partial \mathbf{u}}{\partial X},$$

$$\gamma_{XY} = \frac{\partial \mathbf{u}}{\partial X} \bullet \mathbf{e}_Y + \frac{\partial \mathbf{u}}{\partial Y} \bullet \mathbf{e}_X + \frac{\partial \mathbf{u}}{\partial X} \bullet \frac{\partial \mathbf{u}}{\partial Y}, \quad \text{etc.}$$

- While the kinematics is given in curvilinear co-ordinates, the chain rule for differentiation is needed

$$\frac{\partial}{\partial\alpha}=\frac{\partial X}{\partial\alpha}\frac{\partial}{\partial X}+\frac{\partial Y}{\partial\alpha}\frac{\partial}{\partial Y}+\frac{\partial Z}{\partial\alpha}\frac{\partial}{\partial Z}$$

$$\frac{\partial}{\partial\beta}=\frac{\partial X}{\partial\beta}\frac{\partial}{\partial X}+\frac{\partial Y}{\partial\beta}\frac{\partial}{\partial Y}+\frac{\partial Z}{\partial\beta}\frac{\partial}{\partial Z}$$

$$\frac{\partial}{\partial\gamma}=\frac{\partial X}{\partial\gamma}\frac{\partial}{\partial X}+\frac{\partial Y}{\partial\gamma}\frac{\partial}{\partial Y}+\frac{\partial Z}{\partial\gamma}\frac{\partial}{\partial Z}$$

$$\Rightarrow\begin{Bmatrix}\frac{\partial}{\partial X}\\ \frac{\partial}{\partial Y}\\ \frac{\partial}{\partial Z}\end{Bmatrix}=\begin{bmatrix}\frac{\partial\mathbf{r}}{\partial\alpha}\cdot\mathbf{I} & \frac{\partial\mathbf{r}}{\partial\alpha}\cdot\mathbf{J} & \frac{\partial\mathbf{r}}{\partial\alpha}\cdot\mathbf{K}\\ \frac{\partial\mathbf{r}}{\partial\beta}\cdot\mathbf{I} & \frac{\partial\mathbf{r}}{\partial\beta}\cdot\mathbf{J} & \frac{\partial\mathbf{r}}{\partial\beta}\cdot\mathbf{K}\\ \frac{\partial\mathbf{r}}{\partial\gamma}\cdot\mathbf{I} & \frac{\partial\mathbf{r}}{\partial\gamma}\cdot\mathbf{J} & \frac{\partial\mathbf{r}}{\partial\gamma}\cdot\mathbf{K}\end{bmatrix}^{-1}\begin{Bmatrix}\frac{\partial}{\partial\alpha}\\ \frac{\partial}{\partial\beta}\\ \frac{\partial}{\partial\gamma}\end{Bmatrix}$$

In orthogonal systems the transformation matrix will be diagonal

The kinematics is given in curvilinear geometry

$$\mathbf{u}(\alpha,\beta,\gamma) = u\mathbf{e}_\alpha + v\mathbf{e}_\beta + w\mathbf{e}_\gamma$$

and the derivatives are calculated simply

$$\frac{\partial \mathbf{u}}{\partial \alpha} = \frac{\partial u}{\partial \alpha}\mathbf{e}_\alpha + u\frac{\partial \mathbf{e}_\alpha}{\partial \alpha} + \frac{\partial v}{\partial \alpha}\mathbf{e}_\beta + v\frac{\partial \mathbf{e}_\beta}{\partial \alpha} + \frac{\partial w}{\partial \alpha}\mathbf{e}_\gamma + w\frac{\partial \mathbf{e}_\gamma}{\partial \alpha}$$

All the troubles due to curvilinearity are included in these

Principle of virtual work

- Replaces also the use of the figures of differential geometry
- Represents as an exact mathematical formulation
- Is usable equally well in complicated geometries and non-linear analyses

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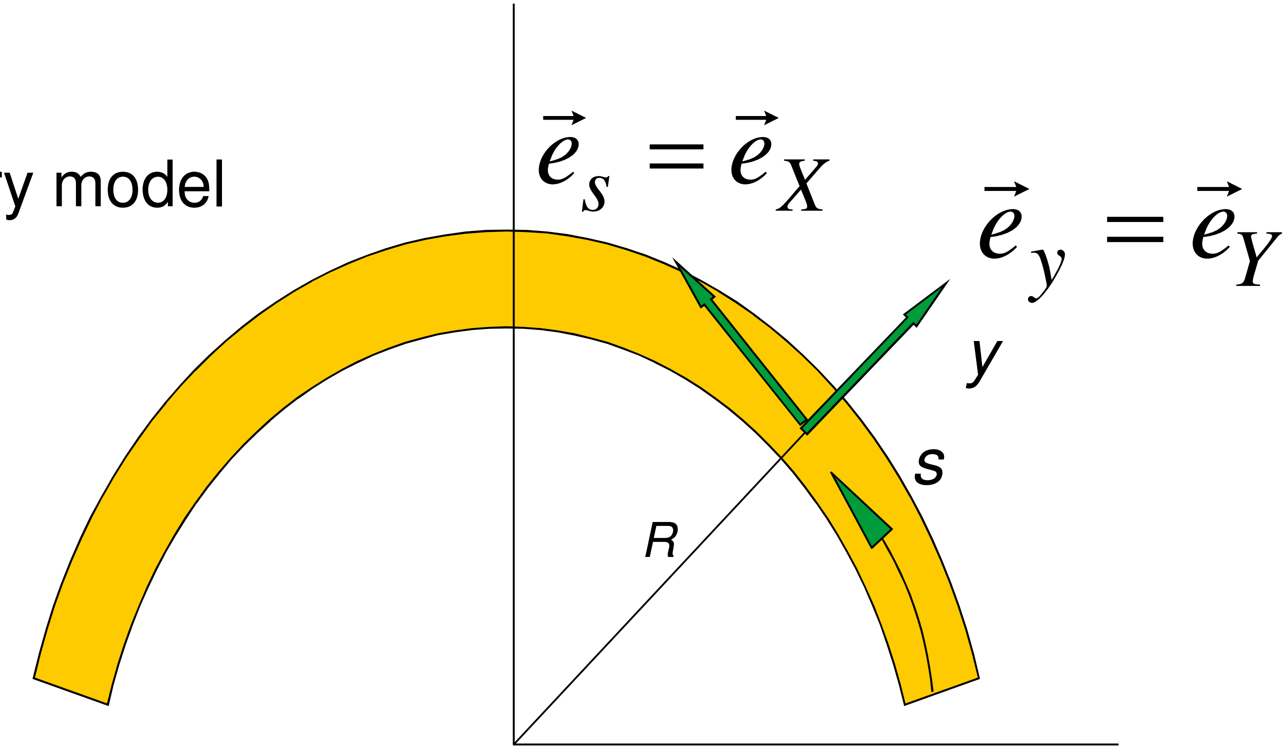
Some Simple Applications

Geometry model



$$\vec{r} = (R + y)\vec{e}_y + z\vec{e}_z$$

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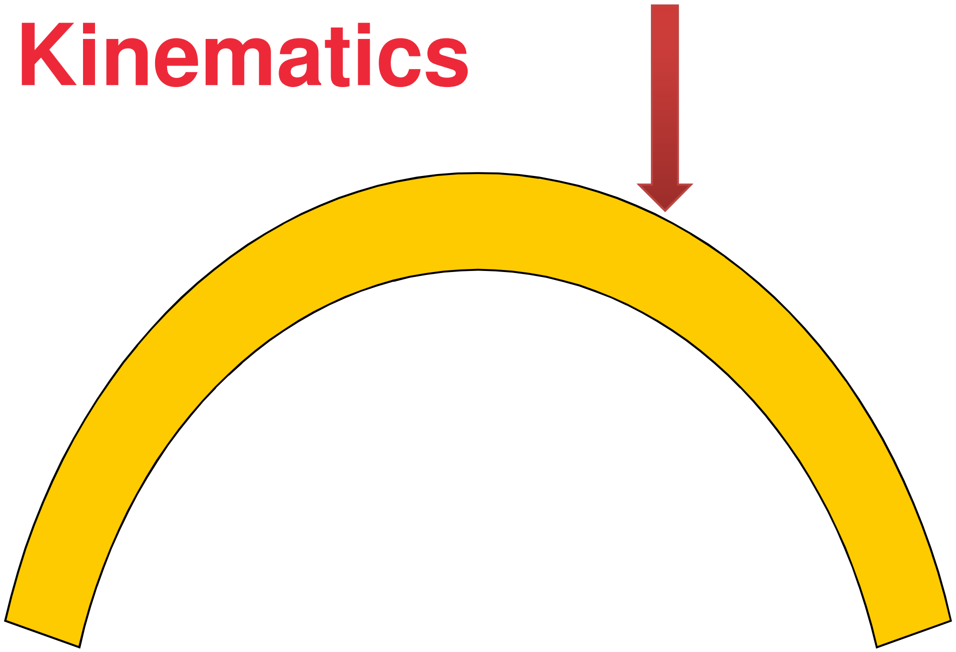
Kinematics



Extension, Bending and Shear

$$\mathbf{u}(s,y,z) = (u - y\theta_z - z\theta_y)\mathbf{e}_s + (v - z\theta_s)\mathbf{e}_y + (w + y\theta_s)\mathbf{e}_z$$

is simplified to $\mathbf{u}(s,y) = (u - y\theta_z)\mathbf{e}_s + v\mathbf{e}_y$

with $u = u(s),\ v = v(s),\ \theta_z = \theta_z(s),\ \mathbf{e}_s = \mathbf{e}_s(s), \mathbf{e}_y = \mathbf{e}_y(s)$

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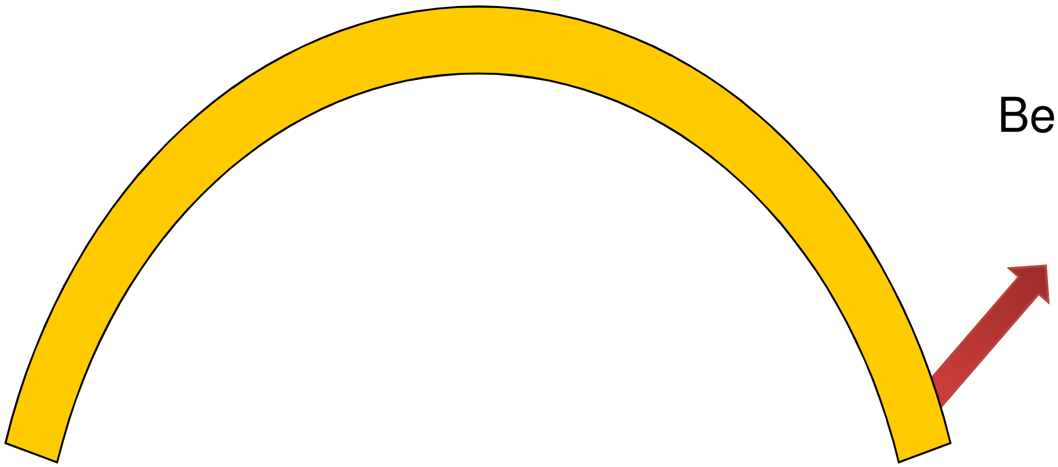
Kinematics

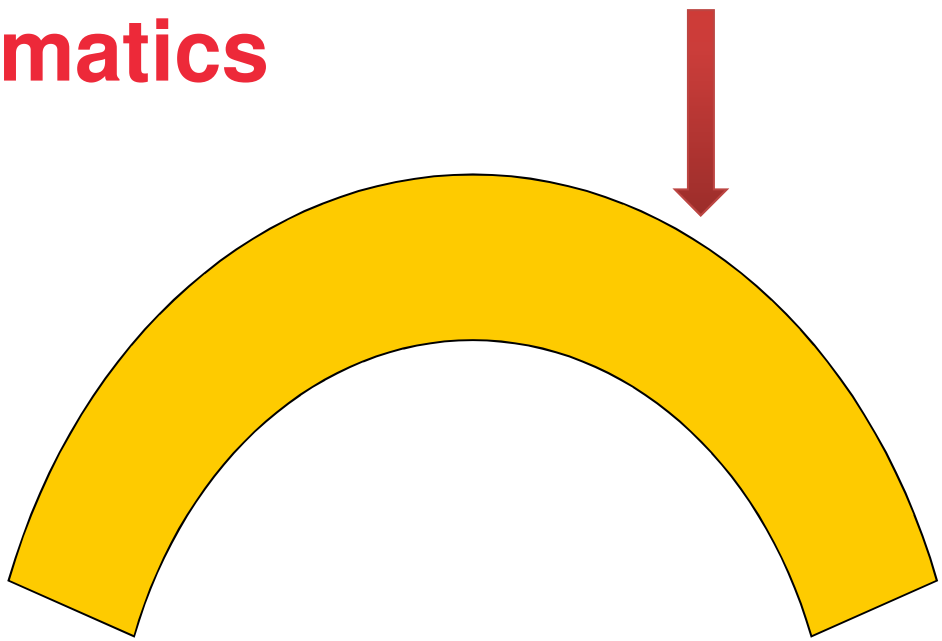


$$\mathbf{u}(s,y,z) = \left(u - y\theta_z - z\theta_y\right)\mathbf{e}_s + \left(v - z\theta_s\right)\mathbf{e}_y + \left(w + y\theta_s\right)\mathbf{e}_z$$

is simplified to $\mathbf{u}(s,y,z) = -z\theta_y\mathbf{e}_s - z\theta_s\mathbf{e}_y + \left(w + y\theta_s\right)\mathbf{e}_z$

$$w = w(s),\ \theta_s = \theta_s(s),\ \theta_y = \theta_y(s),\ \mathbf{e}_s = \mathbf{e}_s(s), \mathbf{e}_y = \mathbf{e}_y(s)$$

Kinematics



Two-dimensional in-plane-bending

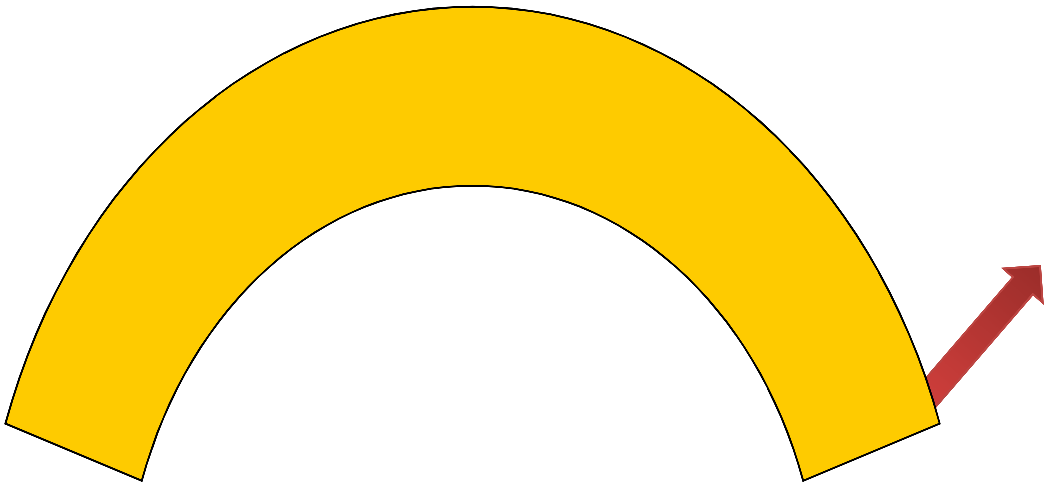
$$\mathbf{u}(s,y) = u\mathbf{e}_s + v\mathbf{e}_y$$

with

$$u = u(s, y),\ v = v(s, y) \text{ and}$$

$$\mathbf{e}_s = \mathbf{e}_s(s, y),\ \mathbf{e}_y = \mathbf{e}_y(s, y)$$

Kinematics



$$\mathbf{u}(s,y,z) = \left[u(s,y) - z\theta_y(s,y)\right]\mathbf{e}_s(s,y)$$
$$+\left[v(s,y) - z\theta_s(s,y)\right]\mathbf{e}_y(s,y) + w(s,y)\mathbf{e}_z$$

Thank you for your time !